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Dynamics of a train of solid particles moving in a channel parallel to a deformable liquid-liquid interface

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This work focuses on the dynamics of a train of solid particles separated by a distance L flowing near the interface of two co-flowing immiscible fluids in a microchannel of height h. Considering that the particle diameter d is of the order of h but much smaller than L, we study numerically how the balance between inertial migration, surface tension and external volumetric forces determine the transverse position of the particles, the shape of the interface separating the two fluids and the surrounding flow structure. Our study includes a systematic analysis of the influence of the governing parameters (fluids viscosity ratio, interface and particle positions, Reynolds Re and capillary Ca numbers and the inter-particle distance L) on the particle equilibrium position. Both inertial and capillary migration are separately discussed. The validity of the linear solution in the limit of pure inertial migration is thoroughly studied in the limit of non-deformable interfaces $Ca = 0$. On the other hand, in the pure capillary regime $Re = 0$, we considered the distinguished limits of very large and very small surface tension. In the later case, the amplitude of the interface deformation induced by the particle is large, comparable to its diameter, but its influence is limited to a small region upstream and downstream of the particle. In the limit of very large surface tension, even though the amplitude of the deformation is small, the region of influence of the interface deformation λ is broad, reaching values of the order of the inter-particle distance L when the surface tension is sufficiently large. This parameter, then, introduces an additional characteristic length that determines the asymptotic behaviour of the flow properties in the limit $Ca^{-1} \ll 1$. The numerical simulations are carried out using the Finite Element Method combined with the Arbitrary Lagrangian-Eulerian (ALE) method to model the deformation of the liquid-liquid interface.

INTRODUCTION

The development of complex fabrication techniques often implies the utilisation of particles embedded in one or both of the fluids that eventually cross the liquid-liquid interface, with encapsulation and coating being some of the most relevant applications [1–4]. In these situations, the couple effects between the hydrodynamic drag, inertial effects, interface deformation and shear or strain near the walls of the channels drive the movement of a particle in a non-quiescent flow towards or away of the fluid–fluid interface at low Reynolds numbers [5]. This paper aims precisely to understand the physical mechanisms that determine the behaviour of a solid particle by calculating the volumetric force $\boldsymbol{f}$ that is necessary to keep the particle stationary between the interface and the channel's walls.

FORMULATION

We consider in this paper two immiscible liquids flowing in a channel with height h. The two liquids form an interface that is located at $y = \Gamma$. The total volumetric flow rate of the two liquids is $Q = Q_1 + Q_2$ and we assume that both liquids have equal density $\rho_1 = \rho_2$ but different viscosity $\mu_1 \neq \mu_2$. A train of particles of diameter d travel at its terminal velocity V' in one of the fluids. Each particle is separated by a distance L' from the closest neighbors, forming the periodic flow structure sketched in figure 1. The transverse location of the particle depends on the intensity of a uniform body force $-\boldsymbol{f'} = -f\boldsymbol{e_y}$ acting on both fluids. To write the problem in non-dimensional form, we chose the channel height h, the average velocity $\bar{u} = Q/h$ and the diffusion time $t_d = \rho_2 h^2/\mu_2$ as the characteristic length, velocity and time used to define the non-dimensional coordinates $\boldsymbol{x} = \boldsymbol{x'}/h$, the dimensionless fluid velocity $\boldsymbol{v}_i = (u_i, v_i) = \boldsymbol{v}_i'/\bar{u}$, pressure $p = p'/(\mu_2\bar{u}/h)$ and time $t = t'/t_d$. The average velocity $\bar{u}$ and the properties of fluid 2 define the Reynolds number of the flow $Re = \rho_2 Q/\mu_2$. Introducing the non-dimensional variables, we obtained the non-dimensional continuity and momentum equations, yielding

$$\nabla \cdot \boldsymbol{v}_i = 0 \quad \text{and} \quad Re\rho\boldsymbol{v}_i \cdot \nabla\boldsymbol{v}_i \quad = -\nabla(p_i + \rho\boldsymbol{f}\boldsymbol{x}) + \nabla \cdot \bar{\bar{\tau}}',$$

with the sub-index referring to the lower $i = 1$ and upper $i = 2$ fluid, respectively, and $\bar{\bar{\tau}}' = \mu(\nabla\boldsymbol{v}_i + \nabla\boldsymbol{v}_i^T)$. The mass force $\boldsymbol{f} = \boldsymbol{f'}/[\bar{u}/t_d]$ is assumed to have only non-zero vertical component $\boldsymbol{f} = f\boldsymbol{e_y}$.

In the reference frame attached to the particle, the velocity of the liquids at the walls $y = 0$ and $y = 1$ is written as $\boldsymbol{v}_i = -V\boldsymbol{e_x}$. Considering that the particle center is located at $x = x_p = 0$, we impose periodicity conditions so that $\boldsymbol{v}(x = -L/2) = \boldsymbol{v}(x = L/2)$ and $p(x = L/2) = p(x = -L/2) - \Delta p$. After imposing μ_1/μ_2, the vertical location of both the particle y_p and the interface $\Gamma = \Gamma_0$ at $x = \pm L/2$, both pressure loss Δp and flow rate Q_1/Q are simultaneously calculated using an iterative method that continues until mass conservation $1 = \int_\Gamma^1 u_2 dy + \int_0^\Gamma u_1 dy$ is satisfied in the whole computational domain. The velocity $V = V'/\bar{u}$ is determined imposing zero force on the particle in x direction.

At the fluid-fluid interface we impose the continuity of velocities $\boldsymbol{v}_1 = \boldsymbol{v}_2$ and the jump condition on the

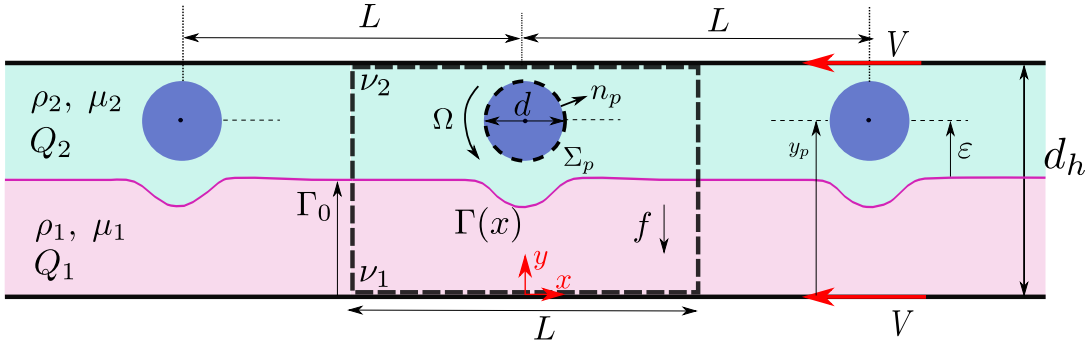

FIG. 1. Schematics of the flow configuration, including the geometrical and fluid-dynamical relevant parameters.

stress tensor $\boldsymbol{n}\cdot(\boldsymbol{T}_1-\boldsymbol{T}_2)=Ca^{-1}\boldsymbol{n}\nabla\cdot\boldsymbol{n}$ with $\boldsymbol{T}_i=-(p_i+\rho\boldsymbol{f}\cdot\boldsymbol{x})\boldsymbol{I}+\mu\left(\nabla\boldsymbol{v}_i+\nabla\boldsymbol{v}_i^T\right)$, $Ca=\mu_2\bar{u}/\sigma$ the capillary number and σ the surface tension.

The non-slip and zero net torque conditions are applied to determine the motion of the rigid particles. The implicit equation $q(x,y,t)=\Gamma(x,t)-y=0$ describes the location of the interface. Because $q=0$ on the interface at all times, the material derivative must satisfy $\boldsymbol{v}\cdot\boldsymbol{n}=0$, with $\boldsymbol{n}=\nabla q/|\nabla q|=(\Gamma_x,-1)/(1+\Gamma_x^2)^{1/2}$ the unit-length vector normal to Γ pointing from fluid 2 towards fluid 1.

CONCLUSION

In this paper we studied the dynamics of a train of solid particles separated a distance L moving near the interface of two co-flowing immiscible liquids. The presence of the interface introduces a force that plays a crucial role to determine the equilibrium position of the particles. The first part of the work considered a non-deformable interface corresponding to the limit of infinitely large surface tension. In this limit, we recover the multiple equilibrium behaviour when the particle is embedded in the less viscous fluid. A maximum of three equilibrium positions is found, two of them stable. When the particle travels in the more viscous fluid, a unique and stable equilibrium position was found regardless of the particle or the interface positions. The asymptotic linear solution for the flow variables $\psi=\psi_0+Re\ \psi_1$ remains valid for values of the Reynolds number as large as $Re=60$ when $\mu_1/\mu_2>1$. This is not the case when $\mu_1/\mu_2<1$ as non-linear effects become relevant even when $Re=0.1$ due to the greater relative importance of the convective terms in fluid 1.
In the pure capillary regime $Re=0$, we computed the mass force necessary to keep the particle at a given vertical location y_p in a wide range of capillary numbers $10^{-4}<Ca^{-1}<10^6$. The results identified three different regimes in the distinguished limits $Ca^{-1}\ll 1$ and $Ca^{-1}\gg 1$ in which the surface tension are very small and very large, respectively. In the former case, the amplitude of the interface deformation induced by the particle is large, comparable to its diameter, but its influence is limited to a small region of the order of the particle size. In this limit, all variables can be expanded asymptotically as $\psi=\psi_0+Ca^{-1}\psi_1$ with the first correction of the mass force written as $Caf=k_3$ and k_3 a constant that depends on the position of both particle and interface. This linear approximation remains valid up to values of the capillary number of order unity as long as $\mu_1/\mu_2>1$. The limit of very large surface tension $Ca^{-1}\gg 1$ turns out to be more complicated. As expected, the amplitude of the interface deformation reduces with increasing Ca^{-1} and becomes much smaller than the particle's diameter. Nevertheless, the region of influence of the particle widens as the surface tension increases, becoming of the order of the inter-particle distance for capillary numbers above a critical value $(Ca^{-1})^*$. Above this value, the asymptotic behaviour of the flow variables become linear with the capillary number so that $\Psi=\Psi_0+Ca^{-1}\Psi_1$ and $f/Ca=k_1$, with k_1 a constant. Below that threshold value $Ca^{-1}<(Ca^{-1})^*$, the relevant characteristic length $\lambda<L$ is determined by the surface tension and the asymptotic behaviour of the solution changes to adopt an asymptotic profile on the form $\psi=\psi_0+Ca^{2/3}\psi_1$ with $f/Ca=k_2Ca^{-1/3}$ and k_2 computed numerically in terms of the parameters of the problem.

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